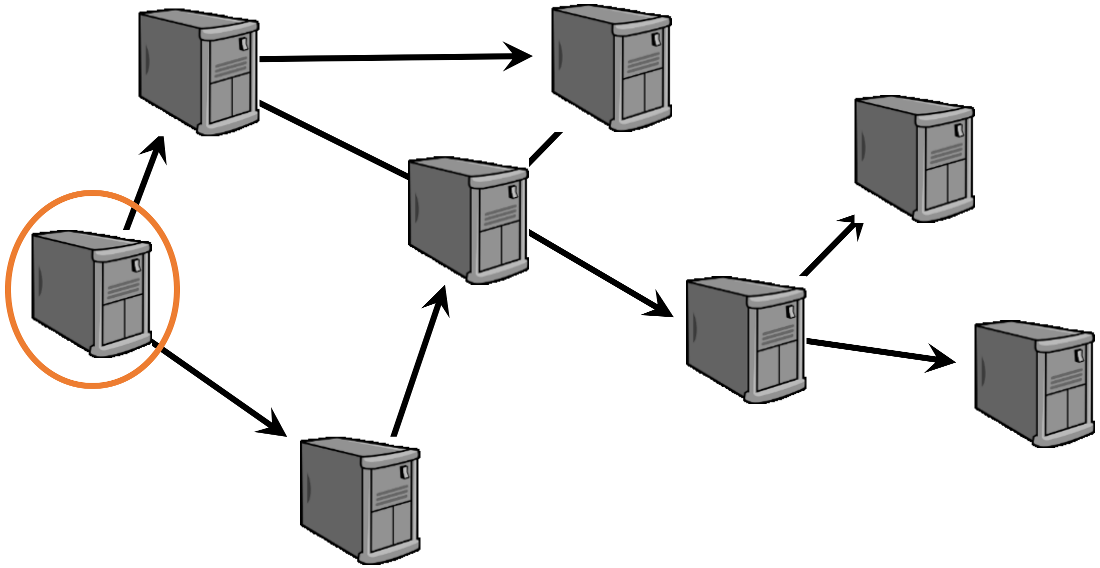
Principals and Practice of Cryptocurrencies

Cornell CS 5437, Spring 2016

Simulation

Example – Gossip

- A messages is generated at one machine
- Message processing time: random 0 - 1sec
- After processing, node chooses random neighbor and sends message
- Sending time: random 0 – 1sec



Why A Simulation?

Formal analysis

Simulation

Proof of concept experiment

Working system experiment

Why A Simulation?

Formal analysis — Simulation — Proof of concept experiment — Working system experiment

Formal Analysis	Simulation	Experiment
Simple models	Complex models	Real world
Basic understanding, parameter interaction	Per-run results – cheap runs	Per-run results – expensive runs
Scale to arbitrary system parameters	Cheap scaling	Expensive scaling

- How complex a model?
    - Over-detailed --> overly complicated
    - Insufficient details --> inaccurate (or wrong)

Event Driven Simulation

Time-driven simulation: second by second

Event-driven simulation: event by event

Which is more accurate?

Event Driven Simulation

Time-driven simulation: second by second

Event-driven simulation: event by event

No tradeoff between efficiency and accuracy

Time	Event
1	Message m arrived at node i
1.3	Message m arrived at node j

Process-Oriented Simulation

- Use an object per entity
- Store individual object state, and let object react to events

Time	Entity	Event
1	node i	Message m arrived
1.3	node j	Message m arrived

Input

Includes both **model details** and **runtime inputs**

- Synthetic, e.g.,
  - Topology of huge system
  - Input arrival times
- Traces, measurement-based, e.g.,
  - Input arrival times
  - Processing times

Output

Data collection:

- Output as much data as possible (probably in a log), but not too much – it can easily explode. For example:
    - Message arrival time at each node:
      Can then calculate 90^{th} percentile propagation time without re-running
    - But not every send / processing-start event

Tips:

- Output meaningful data during long runs
- Output execution parameters in log

Executions

Multiple executions

- Each with different random inputs
- Warmup
  - Do not consider for statistics
  - E.g., when multiple messages are propagated together, let queues stabilize
- Or a single long run, divided to sections

Restart and Avoiding It

- Memoize
  - Carefully while you're debugging…
- Checkpoint
  - For crash handling
  - If you decide continue, to avoid restart

Probability

Probability

- An **experiment** produces a **random results**, ω
- The sample space Ω is all possible results, $\omega \in \Omega$
- An Event is a subset of the sample space, $E \subset \Omega$

Probability

- Every event E has a **probability** $0 \le P[E] \le 1$
- The **conditional probability** of A given B is the probability of A given the B is true; $P[A|B] = \frac{P[A \cap B]}{P[B]}$
- A **random variable** is a function from the sample space to a discrete or continuous range
- A random variable has a **Cumulative Distribution Function** (CDF):

$$F_X(x) = P(X \le x)$$

$$F_X(x) \xrightarrow{x \to -\infty} 0, \qquad F_X(x) \xrightarrow{x \to \infty} 1$$

- A discrete random variable has a probability per value
- A continuous random variable has a **probability distribution function** $f_X(x) = F_X'(x)$

Probability

- The **mean** of a variable X is
$$E[X] = \int_{-\infty}^{\infty} x f_X(x) dx$$
- The **variance** of a variable X is
$$\mathrm{var}[X] = E[(X - E[X])^2] = E[X^2] - (E[X])^2$$

Example – Bernoulli distribution

- Bernoulli distribution:

E.g., due to a biased coin toss, resulting in heads (ω_1) or tails (ω_2)

$$X = \begin{cases} 1 & \text{heads} \\ 0 & \text{tails} \end{cases}$$

$$P[X = 1] = p, P[X = 0] = 1 - p$$

$$E[X] = 1 \cdot p + 0 \cdot (1 - p) = p$$

$$\mathrm{var}[X] = p(1 - p)$$

Example – Normal Distribution

E.g., height, measurement errors

$$N(\mu, \sigma^2)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E[X] = \mu$$

$$\mathrm{var}[X] = \sigma^2$$

The standard normal distribution is $N(0, 1)$.

Example – Normal Distribution

The law of large numbers: The average of Independent and Identically Distributed (IID) random variables converges to the mean of the distribution they are sampled from.

The central limit theorem: the average of IID random variables is approximately normally distributed.

Example – Exponential Distribution

E.g., interval between phone calls

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$E[X] = 1/\lambda$$

$$\text{var}[X] = 1/\lambda^2$$

Memorylessness

For an exponential random variable, $F(x) = 1 - e^{-\lambda x}$:

$$\boldsymbol{P}[\boldsymbol{X} > \boldsymbol{s} + \boldsymbol{t} | \boldsymbol{X} > \boldsymbol{t}] = \frac{P[X > s + t \cap X > t]}{P[X > t]}$$

$$= \frac{P[X > s + t]}{P[X > t]} = \frac{1 - F(s + t)}{1 - F(t)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda t}}$$

$$= e^{-\lambda s} = 1 - F(s) = \boldsymbol{P}[\boldsymbol{X} > \boldsymbol{s}]$$

It doesn't matter how long you have been waiting for the bus

Psuedo Random Number Generator

- No real randomness
- PRNG has state and output
  - State changes on every output request
    (loops, but usually not an issue)
  - Can be reset with given seed

```
>>> import random
>>> random.random()
0.2865104010708595
>>> random.random()
0.179679106469405
>>> random.seed(42)
>>> random.random()
0.6394267984578837
>>> random.random()
0.025010755222666936
>>> random.seed(42)
>>> random.random()
0.6394267984578837
>>> random.random()
0.025010755222666936
```

Use Object PRNG

- Program modular
- Reset global PRNG in each module?
- No – object PRNG per module
- Seed module's PRNG on init

```
>>> randA = random.Random(42)
>>> randB = random.Random(42)
>>> randA.random()
0.6394267984578837
>>> randB.random()
0.6394267984578837
>>> randA.random()
0.025010755222666936
>>> randB.random()
0.025010755222666936
```

PRNG for Simulation

- Different seeds for different runs
- Reproducibility (mostly for debugging)
  - Manually change between runs
  - Seed time, but record seed for reproduction

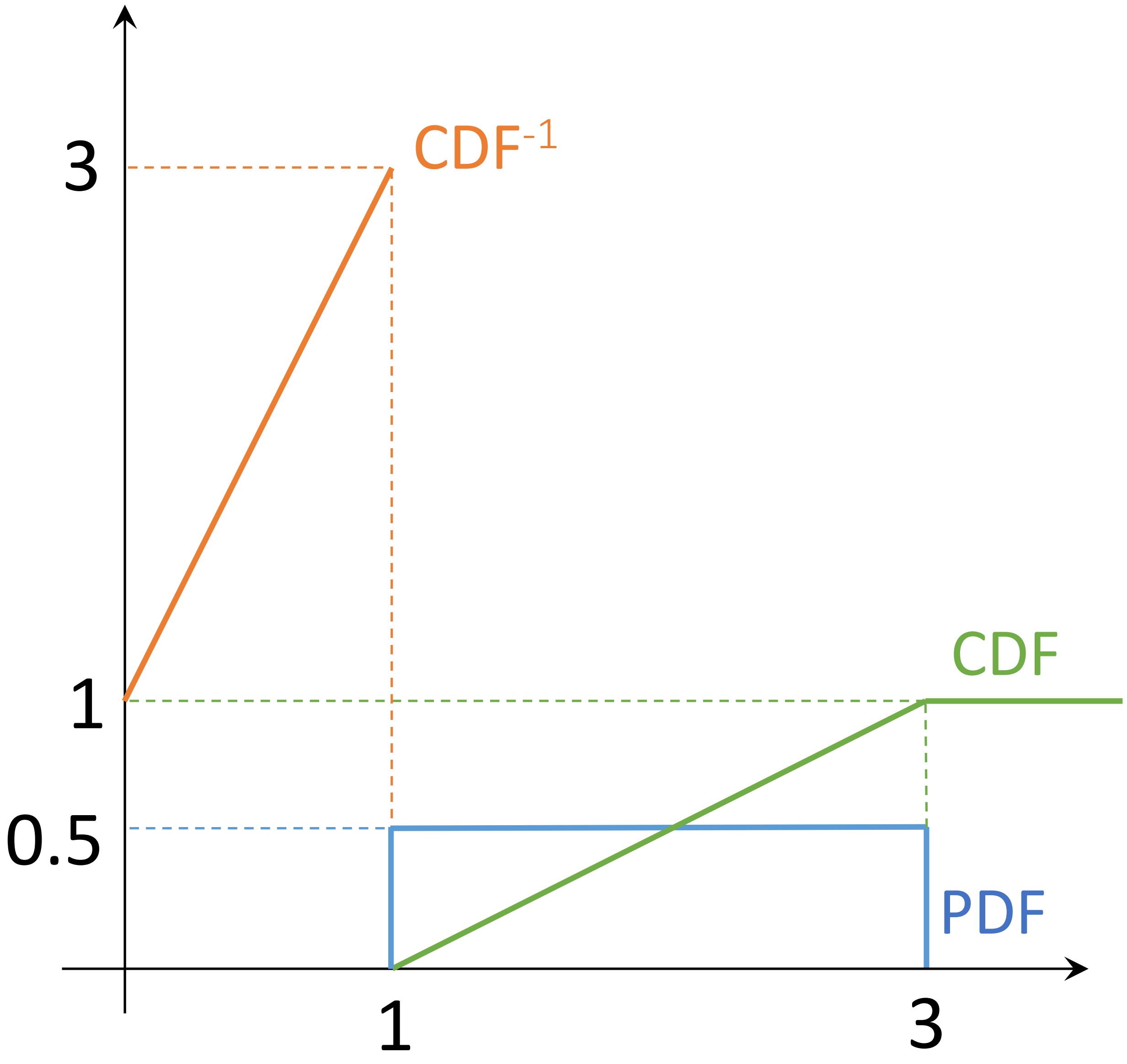
Inverse Transform Sampling

$$f(t) = \begin{cases} 0.5 & 1 \le t < 3 \\ 0 & \text{otherwise} \end{cases}$$

$$F(t) = \begin{cases} 0 & t < 1 \\ 0.5(t-1) & 1 \le t < 3 \\ 1 & t \ge 3 \end{cases}$$

$$F^{-1}(u) = 1 + 2u$$

```
>> randA = random.Random()
>> x = 1 + 2 * randA.random()
```



Statistical Analysis

Given a finite set of measurements

- Estimate the properties of the sampled space
- Estimate the estimation accuracy

Stop when the accuracy is sufficient.

Statistical Analysis

Take a sample of size n $\{x\_i, 1 \leq i \leq n\}$ of independent measurements. E.g., simulation propagation times from n runs.

Sample are taken from a population with probability distribution with mean μ and variance σ^2. (μ and σ are unknown)

- The sample mean is:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- The sample variance is:

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

Statistical Analysis

But the sample mean is a random variable.

The mean of $\bar{x}$ is μ.

So $\bar{x}$ is an estimator of μ

- $\bar{x}$ is not μ, and
- S is not σ

Statistical Analysis

What is the variance of $\bar{x}$?

Statistical Analysis

What is the variance of $\bar{x}$?

$$\mathrm{var}[\bar{x}] = \frac{\sigma^2}{n}$$

More samples --> smaller variance --> $\bar{x}$ probably closer to μ

But we can only take a finite number of samples n.
And we don't have σ, only S.
So we need to estimate σ^2.

Statistical Analysis

What is the mean of S^2?

Statistical Analysis

$$S^2 = \frac{1}{n-1}\sum_{n=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\sum_{n=1}^{n}\left(x_i - \frac{1}{n}\sum_j x_j\right)^2 =$$

$$= \frac{1}{n-1}\sum_{n=1}^{n}\left((x_i - \mu) - \frac{1}{n}\sum_j (x_j - \mu)\right)^2 =$$

$$= \frac{1}{n-1}\sum_i\left((x_i - \mu)^2 + \frac{1}{n^2}\left(\sum_j (x_j - \mu)\right)^2 - \frac{2}{n}(x_i - \mu)\sum_j (x_j - \mu)\right) =$$

$$= \frac{1}{n-1}\sum_i (x_i - \mu)^2 + \frac{1}{n(n-1)}\left(\sum_j (x_j - \mu)\right)^2 - \frac{2}{n(n-1)}\sum_i (x_i - \mu)\sum_j (x_j - \mu) =$$

Statistical Analysis

$$\frac{1}{n-1}\sum_i (x_i-\mu)^2 + \frac{1}{n(n-1)}\left(\sum_j (x_j-\mu)\right)^2 - \frac{2}{n(n-1)}\sum_i (x_i-\mu)\sum_j (x_j-\mu) =$$

$$= \frac{1}{n-1}\sum_i (x_i-\mu)^2 - \frac{1}{n(n-1)}\left(\sum_i (x_i-\mu)^2 + \sum_i \sum_{j\neq i} (x_i-\mu)(x_j-\mu)\right) =$$

$$= \frac{1}{n}\sum_i (x_i-\mu)^2 - \frac{1}{n(n-1)}\sum_i \sum_{j\neq i} (x_i-\mu)(x_j-\mu)$$

$$E[S^2] = \frac{1}{n}\sum_i E[(x_i-\mu)^2] - \frac{1}{n(n-1)}\sum_i \sum_{j\neq i} \cancel{E\left[(x_i-\mu)(x_j-\mu)\right]} = \sigma^2$$

Statistical Analysis

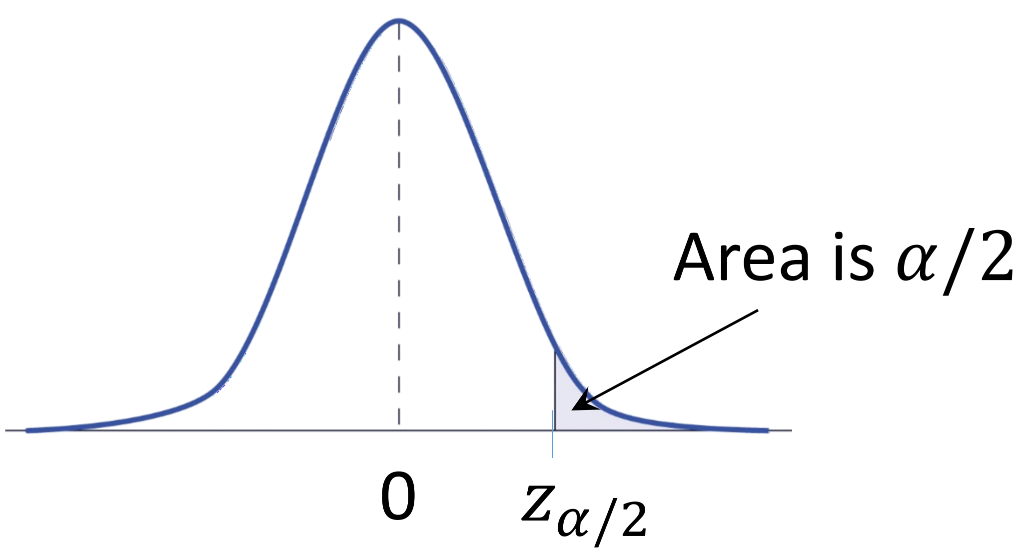
What is the mean of S^2? σ^2.

Confidence Interval

With the standard normal distribution $N(0,1)$, define $z_{\alpha/2}$ to be the point for which the integral to the right is $\alpha/2$.
(tables online)

$$P\left[-z_{\alpha/2} \leq z \leq z_{\alpha/2}\right] = 1 - \alpha$$

For example, for $\alpha = 0.05$, $z_{0.025} = 1.96$



Confidence

So how accurate is the estimate $\bar{x}$?

If the x_i's are normally distributed: $x_i = N(\mu, \sigma)$, let

$$Z = \frac{\bar{x} - \mu}{\sigma}\sqrt{n}$$

Z is normally distributed $Z = N(0, 1)$. So

$$P\left[-z_{\alpha/2} \leq Z = \frac{\bar{x} - \mu}{\sigma}\sqrt{n} \leq z_{\alpha/2}\right] = 1 - \alpha$$

$$P\left[\bar{x} - \frac{z_{\alpha/2}\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + \frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right] = 1 - \alpha$$

For **confidence level** $1 - \alpha$,

the **confidence interval** is $\left[\bar{x} - \frac{z_{\alpha/2}\sigma}{\sqrt{n}}, \bar{x} + \frac{z_{\alpha/2}\sigma}{\sqrt{n}}\right]$

Confidence Interval

- Since we don't have σ, we approximate with S.
- Although the x_i's are not necessarily normally distributed, according to the central limit theorem, it's a good approximation for their sum. Thumb rule: $n \geq 30$.

Example

Index	Value
1	1.9
2	2.0
3	1.9
4	2.0
5	2.1
6	1.9
7	2.1
8	2.1
9	2.1
10	2.1

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 2.05$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = 0.009$$

$$z_{0.05/2} = 1.96$$

$$\bar{x} \pm z_{0.05/2}\frac{S}{\sqrt{n}} = \boxed{(1.99,2.11)}$$

Statistical Analysis

Given a finite set of measurements

- Estimate the properties of the sampled space
- Estimate the estimation accuracy

Stop when the accuracy is sufficient.

E.g., the confidence interval is of length 0.01sec with a confidence level of 95%.

Difficulty and Block Interval

Bitcoin's Block Difficulty

- Hash ($SHA256^2$) of legal block is smaller than a **target**
- Target is a **256bit** value
- Stored as a **32bit** field called **bits** in blocks

Bits: $\mathrm{0x}\underbrace{\mathrm{18}}_{\text{exponent}}\underbrace{\mathrm{0BC409}}_{\text{significand}}$

Target $\mathrm{0BC409} \cdot 2^{8(18-3)}$

Bitcoin's Block Difficulty

- Hash ($SHA256^2$) of legal block is smaller than a **target**
- Target is a **256bit** value
- Stored as a **32bit** field called **bits** in blocks
- Largest target ($\mathrm{target}_{\mathrm{max}}$) is ***defined*** by bits 0x1d00ffff:
  0x00000000FFFF0000000000000000000000000000000000000000000000000000
- Difficulty is defined with respect to the largest target:

$$\mathrm{difficulty} = \frac{\mathrm{largest\ target}}{\mathrm{target}}$$

Bitcoin's Block Difficulty

How long does it take to find a value smaller than

`0x00000000FFFF0000000000000000000000000000000000000000000000000000`

Or simply:

`0x00000000FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF`

?

But wait – the nonce field size…

Bitcoin’s Block Difficulty

How is the target adjusted?

- Once every 2016 blocks (2016/7/24/6 = 2 weeks)
- 2016 blocks ago was t_0
- Last block was t_f
- Total time is $\Delta = t_f - t_0$ seconds
- Difficulty before D_{old}
- Estimate of total hashes calculated: $2016 \times 2^{32} D_{old}$

We want to find D_{new} such that it will take the system 10 minutes on average to find a block. HW

Agenda

- ~~Difficulty calculation~~
- ~~Time to find a block~~
- ~~Difficulty automatic tuning~~
- Minimum of two exponentials and fair mining